Geometry

This chapter includes six groups of templates designed to help students develop a conceptual understanding of polygons, special polygons, right triangles, similar polygons, angles related to parallel lines, reflections, clock related geometry, the Pythagorean theorem, and three dimensional geometry. The templates have been designed to help students visualize many of the concepts and look for patterns.

Polygons

This section begins with a set of templates with similar figures on each template. Using these templates, students can discover the properties that make figures similar and special ratios associated with similar figures. There are two templates for students to use when they classify special triangles and quadrilaterals so they can visually see the relationship between the different types. A template of pattern blocks and two templates of regular polygons with a common length for a side will provide students an opportunity to explore common polygons.

Pythagorean Theorem

This section of templates contains two templates for students to use to discover the equation known as the Pythagorean theorem, by using the area of the three squares drawn adjacent to the three sides of a right triangle. Then there are several templates that will help students develop an understanding for using the Pythagorean theorem in a special triangle. A template of a tangram can help students explore special relationships within an ancient Chinese puzzle.

3-Dimensional Geometry

This section of templates can be used to help develop a student's understanding of many three dimensional figures. There are three templates that engage students in making a connection between nets and their related solid figure. There are three templates that engage students in drawing isometric *3D* drawings of solids. These three templates can be used with connecting cubes to help students visualize the solids. There is one template that can be used when working with pyramids and prisms that have congruent bases and heights.

Parallel and Non-Parallel Lines

This section contains two templates to help students develop an understanding of the angles related to parallel and non-parallel lines when they are cut by a transversal.

Other Geometric Templates

This section contains a variety of other templates that be used in a geometry class. Three templates are included that convert a Communicator® into a geoboard for exploring geometric ideas. There are two templates to help students work with reflecting a figure over a line of reflection to discover the properties of the reflections. There are two clock-related templates for students to discover angles and rates of change associated with the hands of a clock. Finally, there are two templates that can help students develop a geometric proof.

My own ideas...

Polygons

Similar Polygons

There are five different sets of similar polygon templates. Using protractors and the various templates, students can discover that similar polygons have congruent corresponding angles and that the corresponding sides are proportional. In addition, students can study the ratio of the areas for the similar polygons.

Examples:

- Using the *Similar Isosceles Triangles* template, measure the three angles in each triangle and the sides in each triangle using centimeters. What do you notice about the corresponding angles? (They are equal to each other.) Another way to establish that corresponding angles are equal in measure is to use a Communicator® and trace the angle or angles in one triangle and then position those angles on the corresponding angles of other triangles. What do you notice about the corresponding sides? (They are in the same ratio.) Draw a fourth isosceles triangle that is similar to the three given isosceles triangles. (Answers will vary, but should have corresponding angles congruent to those in the given picture and the corresponding sides in a given ratio.)
- Using the *Similar 30° – 60° Right Triangles* template, trace any larger figure and compare it to a smaller figure. How do the sides compare? (1 to 2, 2 to 3, 3 to 4, and 4 to 5) How many of the smallest triangle fit in the next larger triangle? (four) What does this tell you about the area of the two triangles? (The second triangle has an area that is four times the area of the smallest triangle.)

  Compare other pairs of triangles. (The first triangle fits in the third triangle nine times, the first triangle fits in the fourth triangle 16 times, and the first triangle fits in the fifth triangle 25 times.) What does it tell you about the ratio of their areas? (1 to 4, 1 to 9, 1 to 16, and 1 to 25) What does this tell you about the ratio of their sides? (1 to 2, 1 to 3, 1 to 4, and 1 to 5)
- Using the *Similar L Shapes* template with the Communicators®, ask students to compare the corresponding angles. (All angles are right angles and congruent) Ask students to compare the lengths of the corresponding sides and write a ratio for the corresponding sides of consecutive L's? ($\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$) Separate each L into unit squares. Find the area of each L. (3, 12, 27, 48) What is the ratio of the corresponding areas for each pair of consecutive Ls? ($\frac{1}{4}$, $\frac{4}{9}$, $\frac{9}{16}$) How do these sets of ratios compare to the side ratios? (square of the ratios) What does this show you about the ratio of their areas? (If the side in multiplied by a factor of n, then the area is multiplied by n^2)

Classifying Quadrilateral and Triangle

Two classification templates are included so students can classify different types of quadrilaterals and triangles.

Examples:

- Ask students to write down the names for all the different quadrilaterals on a blank Communicator®. (parallelogram, rectangle, square, rhombus, trapezoid, and ordinary quadrilateral) Ask students to write down the definitions for each quadrilateral. (Definitions should describe the number of parallel sides and right angles.)
- Tell students that all quadrilaterals will fit in one of the spaces on this page. Using the *Classifying Quadrilaterals* template, have students place the ordinary quadrilateral in one of the ovals to show its relationship to other quadrilaterals and tell why it was placed in the space. (Inside the first oval because each of the other ovals show properties for special quadrilater-

als.) Ask students to think about the quadrilaterals parallelogram, rectangle, square, and rhombus. Which of these terms is most general and which space would it fit best? (parallelogram and inside the next smallest oval) What is special about all rectangles, squares, and rhombuses? (They are special types of parallelograms.)

Ask students to place these three types of quadrilaterals in the other ovals to illustrate their relationship. (Square inside the overlapped ovals and rectangle and rhombus in other parts of either overlapping oval.) Ask students to explain their reasoning on the placement of these names. (All rectangles, squares and rhombuses are types of parallelograms, therefore they are inside that oval.) Rectangles are parallelograms with right angles, rhombuses are parallelograms with all four congruent sides and squares are special rectangles (All congruent sides) and special rhombuses (All congruent angles).)

Ask students where they would place the trapezoids. This definition varies so it could be placed in one of two locations. If parallelograms are special trapezoids with two sets of parallel sides, then trapezoids can be placed in an oval surrounding the parallelogram oval. If parallelograms are not special trapezoids then the trapezoids could be placed in an oval that fits inside the largest oval, but not overlapping the parallelogram oval. Ask students to look at their ovals and describe what the ovals tell them about the relationship of the various shapes. (Answers will vary on this, but descriptions should illustrate the student's understanding of the various definitions.)

- Ask students to write down the names for all the different triangles on a blank Communicator®. (acute, right, obtuse, equiangular, scalene, isosceles, equilateral.) Ask students to circle the names acute, right, obtuse, and equiangular. Ask students to separate these names into two groups and name the groups. (Triangles based on measurement of angles and triangles based on the measurement of the sides) Ask students to write down the conditions for each name. (Acute triangle: all angles are acute; obtuse triangle: triangle contains one obtuse angle; right triangle: triangle contains one right angle; equiangular triangle: three congruent angles; scalene triangle: no congruent sides; isosceles triangle: at least two congruent sides; equilateral triangle: three congruent sides)
- Tell students that all triangles will fit in one of the spaces on this page. Using the *Classifying Triangles* template, have students place the name scalene triangle in one of the sections to show its relationship to other triangles and tell why it was placed in the space. (Inside the first rectangle because each of the other inside sections show special triangles.)

Ask students to place the names of triangles based on angle measurement in the diagram (acute, obtuse, and right) and describe their placement. (The names acute, right and obtuse can be placed inside the three parts of the middle rectangle in any order because none of these names can be shared by any one triangle.)

- Ask students to place the names of the triangles based on the measurement of the sides in the diagram and explain their reasoning for the placement. (Isosceles triangles can fit in all three sections of the smallest rectangle because an isosceles triangle can be a right triangle, obtuse triangle or an acute triangle.

Ask students to add a section to the picture that illustrates how equilateral (and/or equiangular triangles) are related to the other triangles. (A section can be added that overlaps the acute triangle and isosceles triangle sections because all three sides are congruent and all three angles are congruent and acute.) Ask students to describe what the diagram shows them about different types of triangles. (Answers will vary, but their descriptions should illustrate their understanding of the definitions for each triangle.)

Pattern Blocks

A template of *Pattern Blocks* is included. Tessellations can also be explored with the pattern blocks. Pattern blocks can also be used to develop fractional understanding.

Examples:

- Ask students to find the size of each angle of each pattern block by tracing them adjacent to each other to form 180 degrees (straight line) or 360 degrees.
- Ask students to select three shapes from the pattern blocks except the skinny parallelograms. Ask students to use a total of five of these shapes to form a design. If the design represents one unit, ask students to describe what fractional part of the design each shape is. (Answers will vary.)

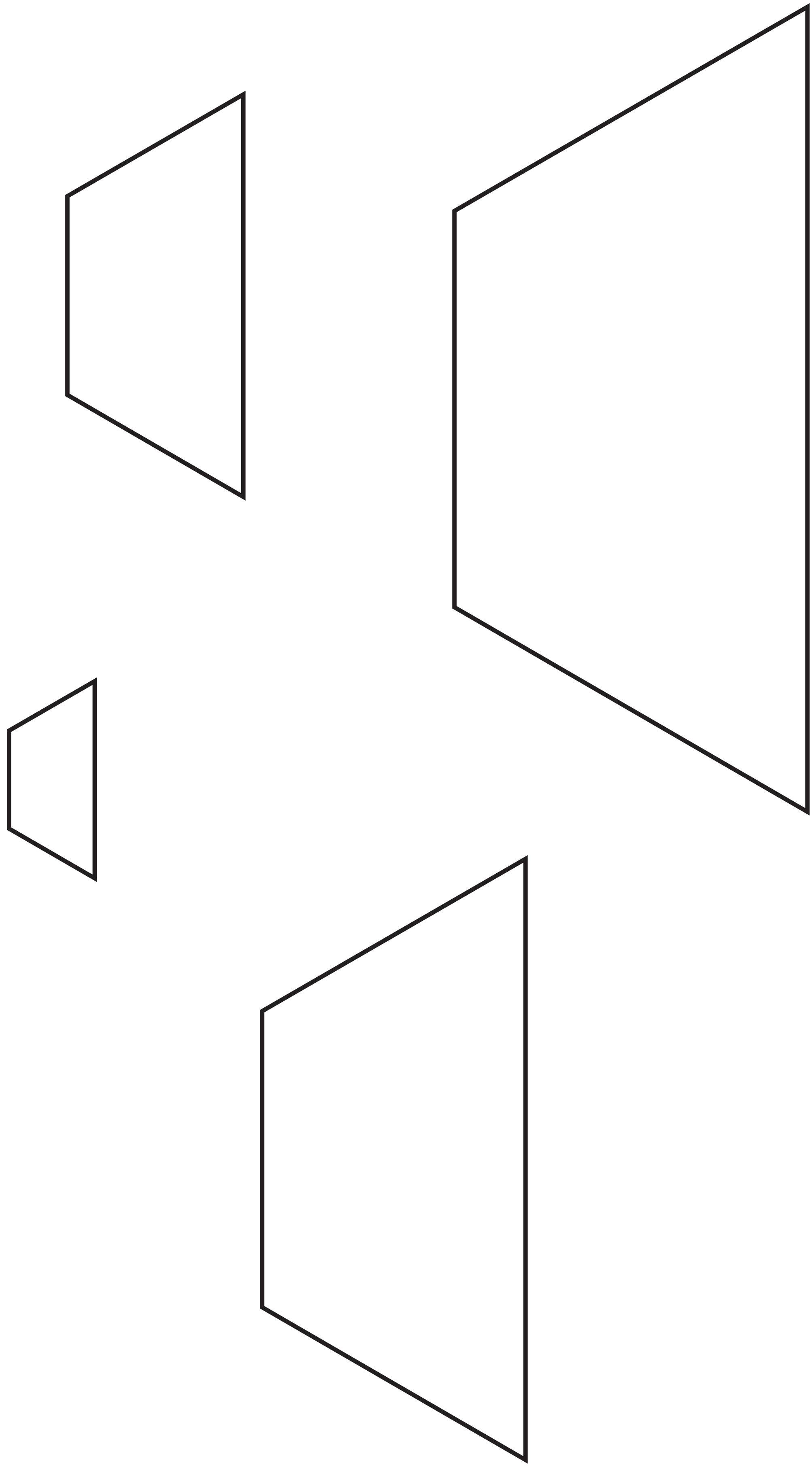
Regular Polygons with Common Length for Sides

There are two templates of regular polygons that can be used to explore tessellations.

- Ask students to place a point on their Communicator® and then place the Communicator® on top of an equilateral triangle on the *Regular Polygon with Common Side 1* template so the point lines up on a vertex. Ask students to continue to trace additional equilateral triangles adjacent to the first triangle to see if they can surround the point with equilateral triangles (yes). This is called tessellating the surface of a plane. Why does the equilateral triangle tessellate the surface? (The angles of the triangle are 60 degrees and 6×60 equals 360 degrees.)
- Ask students to try tessellating the surface of the Communicator® using each of the other polygons. Ask students to describe which shapes tessellate the surface and tell why. (square because $4 \times 90 = 360$ degrees; hexagon because $3 \times 120 = 360$; all others did not tessellate because their one angle is not a factor of 360 degrees)
- Ask students to tessellate the surface with more than one shape (semi-pure tessellation). (There are several semi-pure tessellations such as octagons and squares or hexagons and triangles.)

My own ideas...

Similar Trapezoids



Trigonometry-Precalculus-Calculus

The templates in this chapter can be used to develop concepts in trigonometry, pre-calculus, and calculus classes. There are three sections of templates that cover the topics of trigonometry, derivatives and transforming functions.

Trigonometry

There are several templates that will help students develop their understanding of the unit circle and the trigonometric ratios of sine, cosine, and tangent. Some templates use both degrees and radians while others use one of the measurements.

Derivatives

There are two templates that can be used with students as they work with the function and its derivative. One of the templates gives your students an opportunity to make connections between the graphs of f, f', and f''.

Transforming Functions

There are two templates that can help students work with a base function and using stretching, shrinking, and translations other graphs can be created and new equations can be written.

My own ideas...

Trigonometry

Unit Circle

An *Un-Labeled Unit Circle* template is designed for students to learn to label the key points about the unit circle. You will notice that the coordinate axes intercepts and three points in each quadrant are marked but unlabeled. The un-labeled unit circle also allows for students to use the same template, but change the radius to a different measurement. The *Labeled Unit Circle* template contains both the degree and radian measure and coordinates for each labeled point on a unit circle. A *Degree-Radian Chart* template will give the students a chart to work with the unit circle in setting up a conversion between degrees and radians.

Examples:

- Ask students to place the *Un-Labeled Unit Circle* template in their Communicators®. Ask the students to label the degrees for each of the marked points. (0, 30, 45, 60, 90, 120, 135, 150, 180, 210, 225, 240, 270, 300, 315, 330 and 360 degrees)
- Ask students to label the radian measure for each marked point. (0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, $\frac{2\pi}{3}$, $\frac{3\pi}{4}$, $\frac{5\pi}{6}$, π, $\frac{7\pi}{6}$, $\frac{5\pi}{4}$, $\frac{4\pi}{3}$, $\frac{3\pi}{2}$, $\frac{5\pi}{3}$, $\frac{7\pi}{4}$, $\frac{11\pi}{6}$, and 2π)
- Ask the students to label the radius as 1 unit and then draw in the various right triangles to label the coordinates for each marked point. ((1,0), $(\frac{\sqrt{3}}{2},\frac{1}{2})$, $(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$, $(\frac{1}{2},\frac{\sqrt{3}}{2})$, (0,1), $(-\frac{1}{2},\frac{\sqrt{3}}{2})$, $(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$, $(-\frac{\sqrt{3}}{2},\frac{1}{2})$, (–1,0), $(-\frac{\sqrt{3}}{2},-\frac{1}{2})$, $(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2})$, $(-\frac{1}{2},-\frac{\sqrt{3}}{2})$, (0,–1), $(\frac{1}{2},-\frac{\sqrt{3}}{2})$, $(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2})$, $(\frac{\sqrt{3}}{2},-\frac{1}{2})$, and (1,0))
- Ask students to change the radius of the unit circle to measure two and then ask students to write the coordinates for each of the marked points. (All the answers will be the same as the previous example, but all values will be doubled.)
- Ask students to place the *Labeled Unit Circle* template in their Communicators®. Using the *Degree-Radian Chart* template in conjunction with the *Labeled Unit Circle* template students can discover how radians and degrees are related. Ask students to record the degree and radian measures for the points around the unit circle in the chart. Ask students to look for a pattern and describe it. (180° = π radians)

Trigonometry Graphs

Several templates are included that will help the students develop their understanding of the graphs of the six trigonometric functions (sine, cosine, tangent, cotangent, secant, and cosecant.) The *Unit Circle Graph Generator (Radians)* template and the *Unit Circle Graph Generator (Degrees)* template can be used to help students generate the six graphs from a unit circle. The *Trigonometry Template Positive Degree Measures* template and the *Trigonometry Graph Positive Radian Measures* template can be used by students to graph the six trigonometric functions once they have generated them from the unit circle.

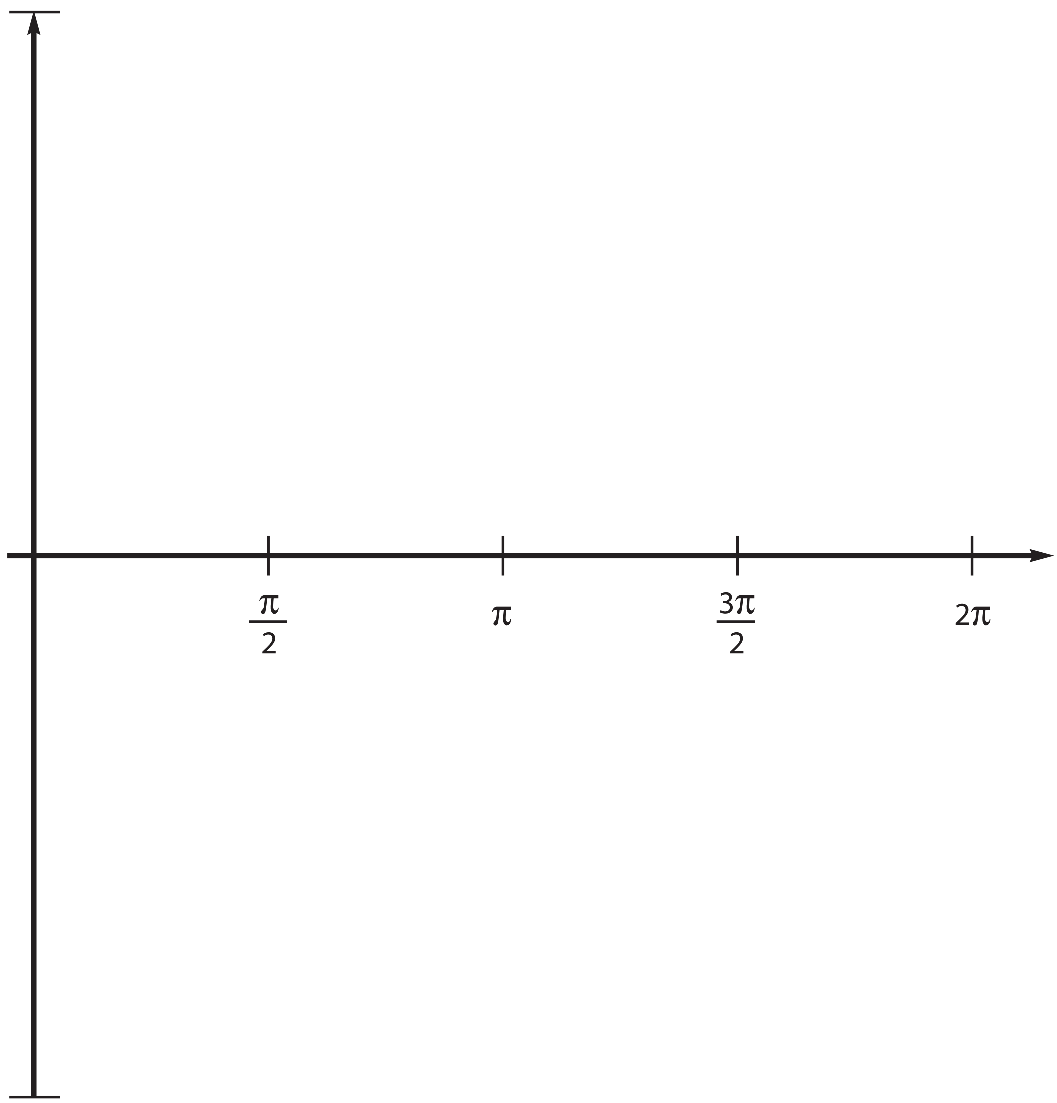
The graphs can be extended into the negative values for degrees and radians by having the students use the *Trigonometry Graph Positive and Negative Degree Measures* template and the *Trigonometry Graph Positive and Negative Radian Measures* template. In addition, there are two templates with multiple copies of the trigonometry templates that show degrees and radians.

Examples:

- Ask students to place the *Unit Circle Graph Generator (Radians)* template in their Communicators®. Explore how the graph of the angle of rotation relates to the *y*-coordinate for each point. For each of the 16 points on the unit circle, ask students to record the coordinates of the point and the angle of rotation. Students should notice that (1,0) can have two different angles of rotation, 0 and 2π, therefore (1,0) will have two different angles of rotation. Then using the graph on the right and information from the unit circle, students should graph 17 new points of the form (*x,y*) where *x* is the angle of rotation for the point and *y* is the *y*-coordinate for the point. (For example, on the unit circle the point ($\frac{\sqrt{3}}{2}, \frac{1}{2}$) corresponds to a rotation of $\frac{\pi}{6}$, therefore the point ($\frac{\pi}{6}, \frac{1}{2}$) would be graphed on the right.) When the students have graphed all 17 points they should draw a smooth curve through the 17 points. (A cosine graph will appear on the graph.) Ask students what they observe about the graph. (The *y* values oscillate between -1 and 1, the *y* values are equal to zero at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, the *y* values are equal to 1 at 0 and 2π, the *y* values is -1 at π, and the graph values are symmetrical around the value of $x = \pi$.)

- Repeat the previous activity with the *Unit Circle Graph Generator (Degrees)* template.
- Ask students to place the *Unit Circle/Graph Generator (Radians)* template in their Communicators®. Again ask students to record the coordinates of the point and the angle of rotation for each of the 16 points on the unit circle. Students should notice that (1,0) can have two different angles of rotation, 0 and 2π, therefore (1,0) will have two different angles of rotation. Then using the graph on the right and information from the unit circle, students should graph 17 new points of the form (*x,y*) where *x* is the angle of rotation for the point and y is the *x*-coordinate for the point. (For example, on the unit circle the point ($\frac{\sqrt{3}}{2}, \frac{1}{2}$) corresponds to a rotation of $\frac{\pi}{6}$, therefore the point ($\frac{\pi}{6}, \frac{\sqrt{3}}{2}$) would be graphed on the right.) When the students have graphed all 17 points, they should draw a smooth curve through the 16 points. (A sine graph will appear on the graph.) Ask students what they observe about the graph. (The *y* values oscillate between -1 and 1 again. The *y* values are equal to zero at 0, π, and 2π. The *y* values are equal to 1 at $\frac{\pi}{2}$. The y value is -1 at $\frac{3\pi}{2}$, and the graph values are not symmetrical around any value of *x*.)
- Repeat the previous activity with the *Unit Circle Graph Generator (Degrees)* template.
- Ask students to place the *Unit Circle Graph Generator (Radians)* template in their Communicators®. Again ask students to record the coordinates of the point and the angle of rotation for each of the 16 points on the unit circle. Students should notice that (1,0) can have two different angles of rotation, 0 and 2π, therefore (1,0) will have two different angles of rotation. Then using the graph on the right and information from the unit circle, students should graph 17 new points of the form (*x,y*) where *x* is the angle of rotation for the point and y is the quotient of the *x*-coordinate and *y*-coordinate for the point. (For example, on the unit circle the point ($\frac{\sqrt{3}}{2}, \frac{1}{2}$) corresponds to a rotation of $\frac{\pi}{6}$, therefore the point ($\frac{\pi}{6}, \sqrt{3}$) would be graphed on the right.) As the students graph the 17 points, they should notice that there were two points where the *y*-coordinate was undefined. ($\frac{\pi}{2}$ and $\frac{3\pi}{2}$) Ask students to begin with the first four points and draw a smooth curve through them. Then skip over the five point and draw a smooth curve through the next eight points. Skip over the 13 point and draw a smooth curve through the next five points. Ask students to think about what the graph will look like between the fourth and fifth points, the fifth and sixth points, the twelfth and thirteenth points, and the thirteenth and fourteenth points. (The curve graphs should approach positive and negative infinity.) Ask students what they observe about the graph. (This graph is quite different from the other two graphs. It does not oscillate between two values as the last two graphs did. This graph also appears to go off to infinity twice, once at $\frac{\pi}{2}$ and again at $\frac{3\pi}{2}$.)
- Repeat the previous activity with the *Unit Circle Graph Generator (Degrees)* template.

Trigonometry Template
Positive Radian Measures



Assessment, Games, and Other Templates

The *Bingo Board 25 Squares* and the *Bingo Board 36 Squares* templates can be used by the teacher to design a specialized review activity. The two bubble sheet templates (*10-Question Answer Sheet, Bubble-in Five Choices,* and the *25-Question Answer Sheet, Bubble-in Five Choices)* can be used by students in their Communicators® to record responses to a teacher-made review or assessment. As students complete their choices ask them to turn over the Communicators®. Then ask the class to reveal their answer to the questions at the same time. By looking around the room, the teacher can quickly assess the responses given by the students and decide the next step that needs to take place.

Three discovery templates are included that ask students to compare and contrast the graphs for differences and similarities. The *Linear Function Discovery Sheet* template gives sets of four linear functions to observe. The graphs change the slope and y-intercept of linear equations. The *Quadratic Function Discovery Sheet* template presents the students with sets of four quadratic functions to observe. Students can describe the different transformations that have taken place. The *Sine Function Discovery Sheet* template presents the students with the base function of $y = \sin x$ and nine graphs that have changed the amplitude and period of the base functions. Students can be asked to describe the changes and write equations for the altered functions.

Examples:

- Ask students to place the *Linear Function Discovery Sheet* template in their Communicators®. Ask students to describe how the graphs in problem set 1, 2, 3, and 4 are the same and how they are different.

Set	Similarities	Differences
1	Same slope	Different y-intercepts
2	All positive slopes. All pass through the origin.	A has slopes of 1 and higher. B has slopes of 1 and less.
3	All negative slopes. All pass through the origin.	A has a slope less steep than or equal to 1. B has a slope steeper than or equal to 1.
4	Same y-intercepts	Different slopes

- Ask students to compare any two graphs and describe their similarities and differences.
- Ask students to place the *Quadratic Function Discovery Sheet* template in their Communicators®. Ask students to look at each problem set and tell how A and B are similar to and different from each other.
- Ask students to compare any two graphs and describe their similarities and differences.
- Ask students to place the *Sine Function Discovery Sheet* template in their Communicators®. Ask students to describe how each of the nine graphs are different and the same as the base function. Ask students to write the equation for each of the nine transformed graphs.

Four sets of matching templates can be used to help students connect the numerical (or tabular), graphical, and analytical form of linear and quadratic equations. The *Linear Tables and Graphs Matching* template asks students to analyze tables and graphs that represent linear functions and look for characteristics represented by both. Similarly the *Linear Tables and Equations Match* template asks students to relate equations and equations that represent linear functions. The *Quadratic Function and Tables Match* template asks students to study equations and tables that represent quadratic functions and to look for common characteristics. The

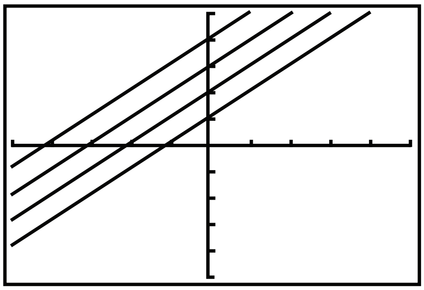
Quadratic Tables and Graphs Match template similarly asks students to relate tables and graphs that represent quadratic functions and look for common characteristics.

Examples:

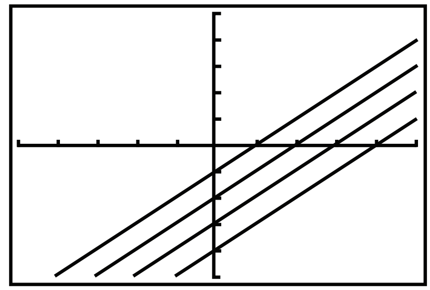
- Ask students to place the *Linear Tables and Graphs Match* template in their Communicators®. Ask students to study the four graphs and four sets of table values. Ask students to choose the tables and graphs that represent increasing functions and explain their reasoning. (Table 1 because as x increases y also increases and Table 4 because as x decreases y decreases. Graphs B and C are increasing because as x increases y increases also, causing the graph to rise to the right.)
- Ask students to match the graph and table that represent the same function and tell why they match. (Table 1 and Graph C, Table 2 and Graph A, Table 3 and Graph D, Table 4 and Graph B)
- Ask students to write the equation for each pair of table and graph pair. (Graph A: $y = -x - 2$, Graph B: $y = x - 2$, Graph C: $y = x + 2$, Graph D: $y = -x + 2$)
- Ask students to place the *Linear Table and Equation Match* template in their Communicators®. Ask students to find all functions that have positive slope. (Find the slope of these functions.) (Tables 2 and 3, x and y are either both increasing or decreasing; Equations A and B shows a positive slope of 2.) What are the slopes on the other functions? (-2) Ask students to match all functions that show a positive y-intercept and determine the y-intercepts. (Tables 3 and 4; equations B and D, y intercept is 1) What are the y-intercept on the other functions? (-1)
- Ask students to put the *Quadratic Functions and Tables Match* template in their Communicators®. Ask students to match each equation with the table set of values that matches that equation. Table 1 and Equation C, Table 2 and Equation A, Table 3 and Equation B, Table 4 and Equation D.
- Ask students to put the *Quadratic Tables and Graphs Match* template in their Communicators®. Ask students to match the graph with the appropriate set of table values. Ask students to support their reasoning. (Table 3 is Graph C. Table 1 is Graph B. Table 2 is Graph D. Table 4 is Graph A. Students can support their answer by using the zeros of the function and whether the values indicate an upright parabola or an upside down parabola.)

My own ideas...

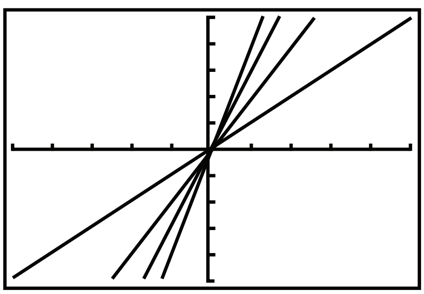
Linear Function Discovery Sheet



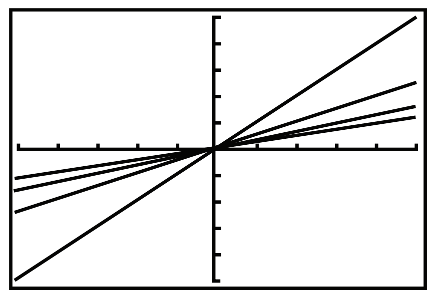
Problem Set 1A



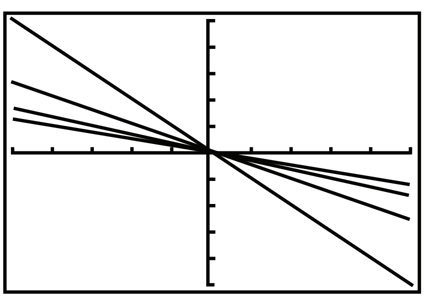
Problem Set 1B



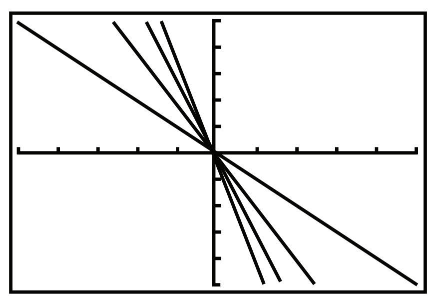
Problem Set 2A



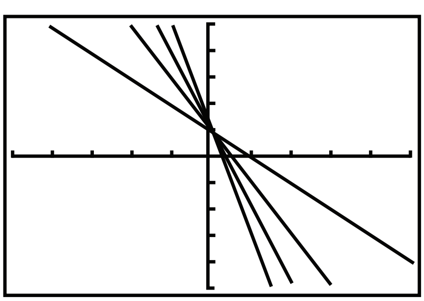
Problem Set 2B



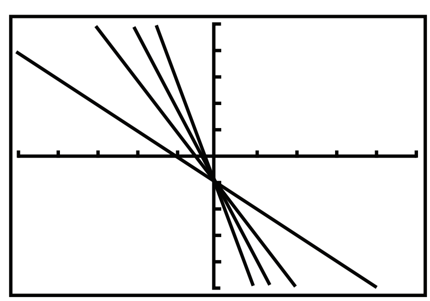
Problem Set 3A



Problem Set 3B



Problem Set 4A



Problem Set 4B

Graph Paper and Grids

This chapter contains three sections of templates. There is a section of coordinate graph paper, a section of dot papers, and a section of other types of graph paper.

Coordinate Graph Paper

This chapter contains templates of various types of graph paper that are useful for graphing and other types of exploration. There are four unlabeled grids, with no axes based on inches (1 inch grid, 1 inch grid with $\frac{1}{2}$ inch subdivisions, 1 inch grid with $\frac{1}{4}$ inch subdivisions, and $\frac{1}{4}$ inch grid) and two unlabeled grids based on centimeters(centimeter grid and centimeter grid with half centimeter subdivisions).

In addition to these grids there are also seven coordinate grid templates. The *First Quadrant Square Coordinate Graph, 0 to 10* template can be used to graph in only the first quadrant. Then the *Four-Quadrant Coordinate Graph Un-labeled Axes ($\frac{1}{4}$" squares)* template, the *Four-Quadrant Coordinate Labeled Graph ($\frac{1}{4}$" squares) Vertical* template, and the *Four-Quadrant Coordinate Labeled Graph ($\frac{1}{4}$" squares) Horizontal* template can be used if you prefer to work with a smaller grid. The *Four-Quadrant Coordinate Un-labeled Graph ($\frac{1}{2}$" squares)* template, the *Four-Quadrant Coordinate Labeled Graph ($\frac{1}{2}$" squares) Vertical* template and the *Four-Quadrant Coordinate Labeled Graph ($\frac{1}{2}$" squares) Horizontal* template are similar to the previous templates, but with larger grids.

Two other coordinate graph templates are included. The *xy Table with Graph* template can be used to help students see relationships between a graphical and numerical representation for a function. The *Multiple 4 Quadrant Graphs* template to provide students smaller grids to compare up to four functions.

Dot Paper

This chapter also contains a set of dot papers that can be used for graphing, drawing polygons and polyhedrons, and other explorations. There are four types of square dot paper templates: the *One Centimeter Dot Paper*, the *Half Centimeter Dot Paper*, the *One Inch Dot Paper*, the *Half Inch Dot Paper*, and the *Quarter Inch Dot Paper*. You will also find two different size isometric dot papers: the *Centimeter Isometric Dot Paper* and the *Two Centimeter Isometric Dot Paper*.

Other Graph Paper

Semi Logarithmic Graph Paper: Semi logarithmic paper has two different scales. One axis has a logarithmic scale and one has a linear scale. Using semi logarithmic paper along with the properties of logarithms can help the students see an application of how logarithms can be applied in real problems. Exponential functions graphed on square graph paper take on certain characteristics. But when these same functions are graphed on semi logarithmic graph paper, the characteristics are changed.

Examples:

- Ask students to write the function $y = 2^x$ on their Communicators®. Ask students to create a table of values for the function $y = 2^x$. (x: 0, 1, 2, 3, 4, 5; y: 1, 2, 4, 8, 16, 32) Ask students to graph this set of points on a coordinate grid. (Graph will be exponential.) Ask students to calculate the logarithm of each *y* value. (log 1 = 0, log 2 = .30, log 4 = .60, log 8 = .90, log 16 = 1.20, log 32 = 1.51)

  Ask students to use a coordinate grid and graph (*x*, log *y*). Ask students to label their *y* coordinates in the form log *y* and its equivalent value (such as log 8 = .90). (The graph should appear linear.) Explore why this happened. Ask students to start with the equation $y = 2^x$ and use logarithms to write the logarithms of both sides. ($\log y = \log 2^x$). Ask students to use the rules of logarithms to simplify their equation. ($\log y = x \log 2$ or $\log y = (\log 2)x$) Ask students to replace log y with the variable Y and rewrite their last equation. ($Y = (\log 2)x$)

Ask students what they notice about this new equation. (It is a linear equation in x and Y with a slope of log 2.) Ask students to select several points on this new graph (x, log y) and calculate the slope of the line. (One example might be ($\frac{\log 2^5 - \log 2^4}{1}$ or $\frac{\log \frac{2^5}{2^4}}{1}$ or log 2.) Ask students to compare this with their modified equation. ($Y = (\log 2)x$ indicates that the graph is a linear equation with a slope of log 2.)

- Ask students to place the *Semi Logarithmic Graph Paper* template in their Communicators®. Ask students to first notice that the horizontal axis is equally spaced, and the vertical axis is scaled differently. The vertical axis is scaled logarithmically. Notice that the sets of lines are in groups of nine unevenly divided segments. Students can label these sections as powers of 10. (1, 10, 100, or .01, .1, 0, 1, 10, etc. depending upon the values they need to graph.)

Ask students to begin at the bottom of the vertical scale with 1 and label the groups 10, 100, etc. Now ask students to graph the set of points for $y = 2^x$. Ask students to label the vertical axis using logarithms. (Label 8 as log 8) Ask students what they notice. (The line appears to be linear.) Ask students to select several points on this new graph (x, log y) and calculate the slope of the line. (One example might be $\frac{\log 16 - \log 8}{1} = \frac{\log 2^4 - \log 2^3}{1}$ or $\frac{\log \frac{2^4}{2^3}}{1}$ or log 2)

Ask students to find the y-intercept for their graph. (0 or the log 1) Ask students to compare the slope and y intercept for the graph with the modified equation. ($Y = (\log 2)x$ indicates that the graph is a linear equation with a slope of log 2, and a y intercept of 0.) Ask students what it means if a graph appears to be linear when graphed on a semi logarithmic graph template. (The originaldata is exponential.)

- Ask students to graph three equations using the *Semi Logarithmic Graph Paper* template: $y = 2^x$, $y = 3^x$, and $y = 4^x$ on the same graph. Ask students what they observe about the graph. (Each line passes through (0, 1). Each line has a different slope.) Ask students to find the slope of each line (log 2, log 3, and log 4). Ask students how they could use this intercept and slope to write an equation of the straight line. ($y = (\log 2)x$, $y = (\log 3)x$, and $y = (\log 4)x$) Ask students to observe the relationship between the exponential equation and the linear equation. What do they notice? (If a linear equation is $y = (\log a)x$, then the exponential equation is $y = a^x$.)
- Ask students to write the equation $y = a \bullet b^x$ on a blank Communicator®. Ask students to take the logarithm of both sides of the equation ($\log y = \log(a \bullet b^x)$). Ask students to use the rules of logarithms to simplify the equation. ($\log y = \log(a) + \log b^x$ or $\log y = \log(a) + (\log b)x$)

Ask students to replace log y with Y. ($Y = \log(a) + (\log b)x$) Ask students to notice that log a and log b are constants. Ask students what they notice about the new equation. (Y is a linear function of x with a slope of log b and a y-intercept of log a.)

- Ask students to graph the following data using the *Semi Logarithmic Paper* template.

x	0	1	2	3	4	5
y	3	6	12	24	48	96

As students are graphing the point (0,3) on semi logarithmic paper, they should realize that it is equivalent to (0, log 3) because of the scaling on the y-axis. Although the data is not linear, when the data is graphed on semi logarithmic paper, the graph is linear. Ask students to find the slope of the line and the y intercept. (The slope will be $\frac{\log 6 - \log 3}{1} = \log \frac{6}{3} = \log 2$ and the y intercept is (0, log 3). Ask students to write the equation for the straight line on the semi logarithmic paper. ($\log y = (\log 2)x + \log 3$) Ask students to simplify this equation using the rules for logarithms to find the exponential equation for the data. ($\log y = \log 2^x + \log 3$ or $\log y = \log(2^x)(3)$ or $y = 3 \cdot 2^x$)

Polar Graphs: Two polar graph templates are included so students can explore graphing on a graph other than the coordinate axis. The *Polar Coordinate Graph Paper – 15° Rays* template shows the various concentric circles and subdivision of 15°. The *Polar Coordinate Graph Paper – 45° Rays* template shows only the four graphing lines, but has subdivisions along the four lines.

Examples:

- Ask students to place the *Polar Coordinate Graph Paper–15° Rays* template and the *Polar Coordinate Chart* template in their Communicators® back to back. Ask students to enter the equation $r = \theta$ on the chart. Using the 16 angles on the unit circle for θ, have the students calculate the values of r. Students should notice that the r and θ are equal. What does this look like on a polar graph? Tell students that they will be graphing points of the form (r, θ). Notice the polar graph paper is made up of rings that represent different values for r and spokes that represent 15 degree (or $\frac{\pi}{12}$ radian) increments. To graph a point (r, θ) a student should start at the center and move along the horizontal line in a positive or negative direction the number represented by r.

  Then students should move along that ring to the spoke representing the value of θ. Ask students to graph (0,0). (A point at the center.) Next ask students to graph ($\frac{\pi}{6}$,) ≈ (0.5, $\frac{\pi}{6}$). (Students should move to the right along the horizontal line to about .5 and then the second spoke which represents $\frac{\pi}{6}$. Ask students to continue graphing additional points from the chart. Students should connect their points with a smooth curve. (A spiral should be created.)
- Ask students to enter the equation $r = 3\sin\theta$ in the right hand column on the chart. Ask students to use the 16 points of the unit circle in the left hand column of the chart. Ask students to evaluate the equation at each value of θ. After students have calculated the values of r, ask students to graph the 16 points. What does the graph look like? (a circle with a radius of 1.5 centered at (1.5, $\frac{\pi}{2}$)
- Ask students to enter the equation $r = 3\sin 2\theta$ in the right hand column on the chart. Ask students to use the 16 points of the unit circle in the left hand column of the chart. Ask students to evaluate the equation at each value of θ. After students have calculated the values of r, ask students to graph the 16 points. What does the graph look like? (A four petal flower that is symmetrical to the x and y axis. Petals have a length of 3 units.)

Trapezoidal Graph Paper: There are two size trapezoidal graph paper templates, each of a different size, that can be used to explore relationships among several different polygons.

Examples:

- Ask students to place the *Inch Trapezoidal Graph Paper* template or the *Half Inch Trapezoidal Graph Paper* template in their Communicator®. Ask students to outline the smallest equilateral triangle on the grid. If the side of this equilateral triangle is equal to 2, ask students to determine the height of the equilateral triangle. ($\sqrt{3}$) (See p. 100 for exploration with an equi-

Four-Quadrant Coordinate Labeled Graph (1/4″ squares) Vertical